A Duplexer Using the Zero Permeability Characteristics of Ferrite*

It has been shown by Polder[1] and others[2] that a ferrite material can be made to exhibit an effective zero permeability to a wave that has a positive sense of circular polarization (positive wave) and a corresponding nonzero permeability to a wave with polarization in the negative sense (negative wave). Melchor, *et al.*,[3] and Duncan and Swern[4] have demonstrated that a ferrite material which is magnetically biased such that the real part of its positive wave permeability is zero will largely exclude the positive wave. The negative wave, however, will be concentrated in the ferrite; and, under certain conditions of ferrite geometry and operating frequency, the negative wave will propagate through the ferrite in a dielectric mode.

This note describes a duplexer which utilizes this differential interaction in ferrite when biased to, or in the region of, zero permeability. Fig. 1 shows a diagrammatic sketch of a duplexer which uses these principles.

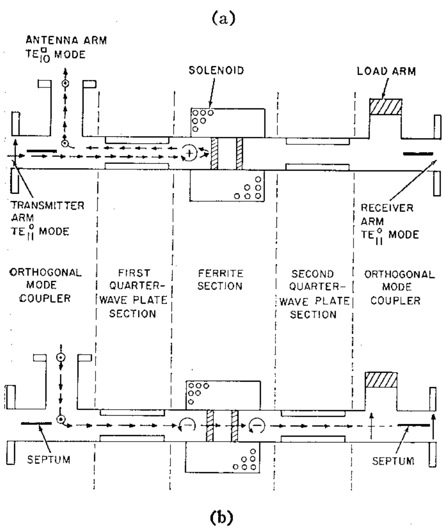

Fig. 1—Diagrammic sketch of a "zero permeability" duplexer. (a) Transmit case. (b) Receive case.

The design of this structure is such that a linearly polarized wave entering the transmitter arm will propagate through the orthogonal mode coupler with negligible coupling to the antenna arm, and will become circularly polarized in the positive sense before entering the ferrite section of the waveguide. The ferrite section consists of one or more ferrite disks which completely fill the cross section of the waveguide as shown in Fig. 1. A solenoid or a cylindrical magnet is used to produce a longitudinal magnetic field to bias the ferrite disks to the region where the real part of the effective ferrite permeability is essentially zero. As discussed in the first paragraph, the positive wave will be almost totally excluded from and, hence, reflected by the ferrite. The reflected wave will propagate through the first quarter-wave plate section which reconverts the wave to linear polarization, oriented 90° with respect to the incident linearly polarized wave. At the orthogonal mode coupler, the wave is then coupled into the antenna arm with negligible losses. The orthogonal mode coupler on the receiver end provides a load for any positive wave energy that is propagated through the ferrite section.

A wave entering the antenna arm will be coupled into the circular waveguide and will propagate into the ferrite section with a negative sense of circular polarization. Since the effective permeability for the negative wave is greater than unity (at the same magnetic bias for zero positive wave permeability), the wave will propagate through the ferrite section if the width and spacing of the disks are such that a proper impedance match is achieved. If the dielectric and magnetic losses of the ferrite material are low, the wave willl be only slightly attenuated when it emerges from the ferrite section. The emergent wave, after propagating through the second quarter-wave plate section, will be reconverted to a linearly polarized wave at the receiver arm oriented perpendicular to the septum in the orthogonal mode coupler. Thus, loss to the load arm is negligible.

Fig. 2 shows an experimental X-band "zero permeability" duplexer. Representative data taken on a similar duplexer designed for 9.2 Gc are given in Fig. 3. The ferrimagnetic material used in this particular unit was yttrium-iron garnet. At 9.2 Gc, the transmitter-to-antenna loss was 0.4 db with a corresponding receiver arm isolation of more than 30 db. The transmitter-to-antenna insertion loss remained well below 1.0 db from 9.0 to 9.7 Gc. Subsequent tests have shown that this loss remains below 1.0 db over a 10 per cent bandwidth. Beyond a 10 per cent bandwidth, the higher loss was due primarily to the narrow bandwidth of the quarter-wave plates used in the experimental work.

The antenna-to-receiver insertion loss at 9.2 Gc was also 0.4 db and remained below 1.0 db over a bandwidth of approximately 5 per cent. Increased loss beyond this bandwidth is due principally to the fixed ferrite geometry and spacing with the narrow-band quarter-wave plates contributing to some degree.

A duplexer of this design offers a definite advantage in that high power effects are minimized since the microwave energy associated with high peak power does not propagate through the ferrite. However, there is some magnetic coupling into the ferrite surface; but, by careful selection of ferrite material and geometry, the magnetic biasing field can be kept well below the field which will give rise to subsidiary resonance losses.

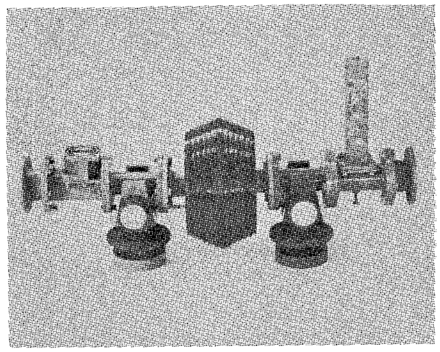

Fig. 2—An experimental X-band zero permeability duplexer.

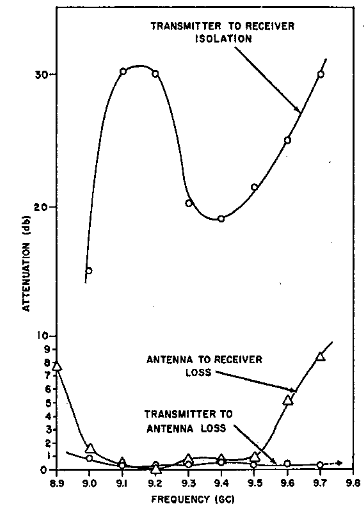

Fig. 3—Characteristics of a zero permeability duplexer using two thin disks of yttrium-iron garnet. $H_{dc} = 925$ gauss.

The duplexer with characteristics shown in Fig. 2 was tested without cooling or pressurization at 250 kw peak (100 w average) at 9.2 Gc with no deleterious effects noted.

Many helpful discussions with Mr. B. J. Duncan, Dr. J. E. Pippin and Dr. G. P. Rodrique, and the help of R. W. Coston in the experimental work presented, are thankfully acknowledged.

L. K. Wilson
G. J. Neumann
Solid State and Microwave
Component Dev. Sec.
Sperry Microwave Electronics Co.
Clearwater, Fla.

* Received by the PGMTT, August 25, 1961. This work has been sponsored by the Wright Air Dev. Div., Dayton, Ohio, under Contract Number AF33-(616)6517.

[1] D. Polder, "On the theory of ferromagnetic resonance," *Phil. Mag.*, vol. 40, pp. 99–114; January, 1954.

[2] See, for example, C. L. Hogan, "The ferromagnetic Faraday effect at microwave frequencies and its applications," *Rev. Mod. Phys.*, vol. 25, pp. 253–263; January, 1953.

[3] J. L. Melchor, W. L. Ayres, and P. H. Vartanian, "Energy concentration effects in ferrite loaded waveguides," *J. Appl. Phys.*, vol. 27, pp. 72–77; January, 1956.

[4] B. J. Duncan and L. Swern, "Effect of zero ferrite permeability on circularly polarized waves," Proc. IRE, vol. 45, pp. 647–655; May, 1957.

A Series-Connected Traveling-Wave Parametric Amplifier*

Introduction

The conventional parametric amplifier consisting of a single variable element with

* Received by the PGMTT, March 1, 1961; revised manuscript received, September 1, 1961. The work reported here was carried out as part of the Lockheed Research Program.

its accompanying resonant circuits is inherently a narrow-band device. Analysis has also shown that such amplifiers when operating at high gain are extremely sensitive to pump power, *i.e.*, small variations in pump power lead to large variations in gain. Traveling-wave parametric amplifiers, in which not one but many variable elements are involved, have been designed or built to overcome the two limitations mentioned above.[1–4] This paper is concerned with the analysis of one such amplifier in which the variable elements are placed in series with the signal line as opposed to one in which the elements are placed in shunt. In particular, the variable elements are capacitance diodes.

With the diodes in series with the signal line, it becomes possible to design amplifiers whose structures are considerably simpler than otherwise. Fig. 1 shows one such design, called a "serpentine" amplifier for obvious reasons. The signal to be amplified is carried by the coax line while the pump wave is carried by the waveguide. Note that now a single pump source rather than multiple pump sources is required.[3] Considering the diodes as representing capacitances shunting the pump line, it is possible to bias and so space them as to allow only the pump frequency component to exist in the pump line. Note also that such an arrangement allows for more flexibility in that the signal line lengths between diodes can be easily adjusted for optimum operating conditions. Such favorable conditions are easily created in the laboratory and they will be assumed in the following discussion.

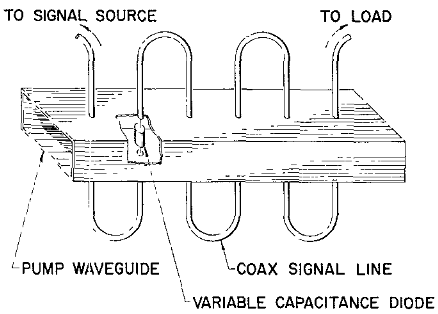

Fig. 1—Serpentine traveling-wave parametric amplifier.

Discussion

The procedure used will follow closely those used by Heffner[1] and Tien.[2] Consequently, a great deal of the intermediate steps will be omitted.

The signal line in Fig. 1 can be represented by Fig. 2 in which L_0 and C_0 are the equivalent series inductance and shunt capacitance per unit length of the line. The line carrying the pump wave is not shown but the effect of the pump is reflected in the elastance $S(t, z)$ of the diode by

$$S(t, z) = S_1 e^{j(\omega_p t - \beta_p z)} + S_1^* e^{-j(\omega_p t - \beta_p z)}, \quad (1)$$

where ω_p is the angular frequency of the pump and β_p is the pump, phase constant. As it turns out, consideration of the elastance rather than the capacitance of series elements proves to be more appropriate with such structures. Note that the actual elastance should contain a constant term S_0 representing the average value. In this paper it is assumed that the value of C_0 is such as to include both the inherent line capacitance and the effects of S_0.

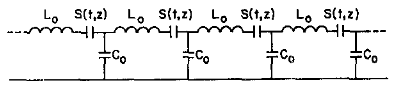

Fig. 2—Signal line.

The differential equations describing the voltage and current relationships over a unit length are

$$\frac{\partial V(t, z)}{\partial z} = -L_0 \frac{\partial I(t, z)}{\partial t} - \int S(t, z) I(t, z) dt \quad (2)$$

and

$$\frac{\partial I(t, z)}{\partial z} = -C_0 \frac{\partial V(t, z)}{\partial t}. \quad (3)$$

Except for the term with the integral sign these are the usual transmission line equations. Parametric action of course results from the new term.

Differentiating (2) with respect to time t gives

$$\frac{\partial}{\partial t}\frac{\partial V(t, z)}{\partial z} = -L_0 \frac{\partial^2 I(t, z)}{\partial t^2} - I(t, z) S(t, z). \quad (4)$$

Differentiating (3) with respect to distance z and interchanging the order of differentiation gives

$$\frac{\partial^2 I(t, z)}{\partial z^2} = -C_0 \frac{\partial}{\partial t}\frac{\partial V(t, z)}{\partial z}. \quad (5)$$

Substituting (4) into (5) leads to

$$\frac{\partial^2 I(t, z)}{\partial z^2} = C_0 L_0 \frac{\partial^2 I(t, z)}{\partial t^2} + C_0 I(t, z) S(t, z). \quad (6)$$

It will be assumed that the current $I(t, z)$ consists only of two relevant components; namely, the signal component designated by $I_1(t, z)$ and the idler component designated by $I_2(t, z)$. Appropriate filter networks are placed in the line to eliminate all other frequency components. Thus

$$I(t, z) = I_1(z) e^{j(\omega_1 t - \beta_1 z)} + I_1^*(z) e^{-j(\omega_1 t - \beta_1 z)} + I_2(z) e^{j(\omega_2 t - \beta_2 z)} + I_2^*(z) e^{-j(\omega_2 t - \beta_2 z)}, \quad (7)$$

where

$$\beta_1 = \omega_1 \sqrt{L_0 C_0} \quad \text{and} \quad \beta_2 = \omega_2 \sqrt{L_0 C_0}.$$

Consider the case where

$$\omega_p = \omega_1 + \omega_2$$

and the lines are such that

$$\beta_p = \beta_1 + \beta_2.$$

Carrying out the operations indicated in (6) and equating the coefficients of like frequency terms leads to a set of four simultaneous equations, of which two are independent of each other. These are

$$\frac{\partial^2 I_1(z)}{\partial z^2} - j2\beta_1 \frac{\partial I_1(z)}{\partial z} = C_0 S_1 I_2^*(z) \quad (8)$$

$$\frac{\partial^2 I_2^*(z)}{\partial z^2} + j2\beta_2 \frac{\partial I_2^*(z)}{\partial z} = C_0 S_1^* I_1(z). \quad (9)$$

The above equations can be further simplified if it is assumed that

$$\left|\frac{\partial^2 I}{\partial z^2}\right| \ll \left|2\beta \frac{\partial I}{\partial z}\right|.$$

Thus

$$\frac{\partial I_1(z)}{\partial z} = -\frac{S_1 C_0 I_2^*(z)}{j2\beta_1} \quad (10)$$

and

$$\frac{\partial I_2^*(z)}{\partial z} = \frac{S_1^* C_0 I_1(z)}{j2\beta_2} \quad (11)$$

or

$$I_1(z) = K_1 \exp\left\{\sqrt{\frac{C_0^2 S_1 S_1^*}{4\beta_1\beta_2}}\, z\right\} + K_2 \exp\left\{-\sqrt{\frac{C_0^2 S_1 S_1^*}{4\beta_1\beta_2}}\, z\right\} \quad (12)$$

and

$$I_2^*(z) = \frac{jS_1^* K_1}{\sqrt{\frac{S_1 S_1^* \beta_2}{\beta_1}}} \exp\left\{\sqrt{\frac{C_0^2 S_1 S_1^*}{4\beta_1\beta_2}}\, z\right\} + \frac{jS_1^* K_2}{\sqrt{\frac{S_1 S_1^* \beta_2}{\beta_1}}} \exp\left\{-\sqrt{\frac{C_0^2 S_1 S_1^*}{4\beta_1\beta_2}}\, z\right\}, \quad (13)$$

where K_1 and K_2 are arbitrary constants. Thus the signal $I_1(z)$ is seen to grow (amplification of the signal) with z. These expressions are similar (but not identical) to those obtained by Heffner[1] and Tien,[2] as expected.

No attempt will be made to compute the bandwidth and noise because it is felt that the results will be similar to those reported by Heffner and Tien.

Conclusions

A traveling-wave parametric amplifier of a rather novel design has been presented. Connecting the variable capacitance diodes in series with the signal line leads to structural and operational simplicity. The analysis proves that under certain favorable conditions amplification of the signal can take place.

Acknowledgment

The author wishes to thank V. Counter for his valuable suggestions and Miss J. Hillhouse and K. Fong for checking the mathematics.

Tuck Hop Lee
Missiles and Space Division
Lockheed Aircraft Corp.
Sunnyvale, Calif.

[1] H. Heffner, "Traveling Wave Parametric Amplifier Analysis," class notes, Stanford University, Stanford, Calif.; 1960.

[2] P. K. Tien, "Parametric amplification and frequency mixing in propagating circuits," *J. Appl. Phys.*, vol. 29, pp. 1347–1357; September, 1958.

[3] C. V. Bell and G. Wade, "Circuit considerations in traveling-wave parametric amplifiers," 1959 IRE WESCON Convention Record, pt. 2, pp. 75–82.

[4] R. C. Honey and E. M. T. Jones, "A wide band UHF traveling-wave variable reactance amplifier," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 351–361; May, 1960.